Drag Coefficient Prediction

Chapter 1

The ideal force acting on a surface positioned perpendicular to the airflow is equal to a dynamic pressure, denoted by '*q*', times the area of that surface. Dynamic pressure is one-half the square of the flow velocity times the density of the fluid. In equation form this is:

$$q = \frac{1}{2} r V^2$$

Where,

$$r = \textit{Air density, lb.-sec}^2 / ft^4$$

$$V = \textit{Flow velocity, ft/sec}$$

Imagine putting your hand or a flat plate out the window of a car such that the flat surface is positioned normal to the airflow. The total force required to hold it in position when meeting the oncoming airflow will be approximately equal to that defined above. Realistically, the actual force is dependent on the shape of the object, and the 3-dimensional flow characteristics of the fluid (i.e.; flow relief, turbulent and/or laminar flow, etc.). Often these influences are summed up in a single coefficient known as an aerodynamic coefficient. In our particular case, we are interested in a drag force '*D*', and likewise a drag coefficient 'C_D'.

Depending on equation formulation and reference area, C_D can take on different values. For rockets, C_D is typically based on the rocket's maximum cross-section area. Most rockets are circular in cross-section, therefore its cross-section area is described by the equation for an area of a circle. In general, the equation for drag '*D*' is given by:

$$D = qC_D A = qC_D p R^2$$

Where,

$$q = \textit{Dynamic pressure, lb.} / ft^2$$

$$C_D = \textit{Drag coefficient}$$

$$A = \textit{Reference area, } ft^2$$

$$= pR^2$$

$$p = \textit{Constant Pi} \text{ » } 3.14159$$

$$R = \textit{Radius of maximum cross-section, ft}$$

Most of the equations presented in this chapter are empirically based (based on physical data). The majority of the equations will be simply presented and not derived, particularly those describing **Friction Drag**. Others, such as those describing **Base Drag** and **Wave Drag**, will be derived as we go along. I believe that the source for the basis of most of these equations is the U.S Air Force methodology known as DATCOM (Reference 4). Many of these equations have been transcribed from hand written notes that I had used in my past career as an aircraft conceptual designer with the U.S. Air Force, many moons ago. The graphs of these notes have been transformed to equations for ease of writing computer programs. I have deviated somewhat from the DATCOM methods for Base Drag estimation, and have developed an approach that fits rocket data more accurately. A constant 'K_F ' equal to 1.04 has been adopted to estimate the **Interference Drag** contribution for rocket data.

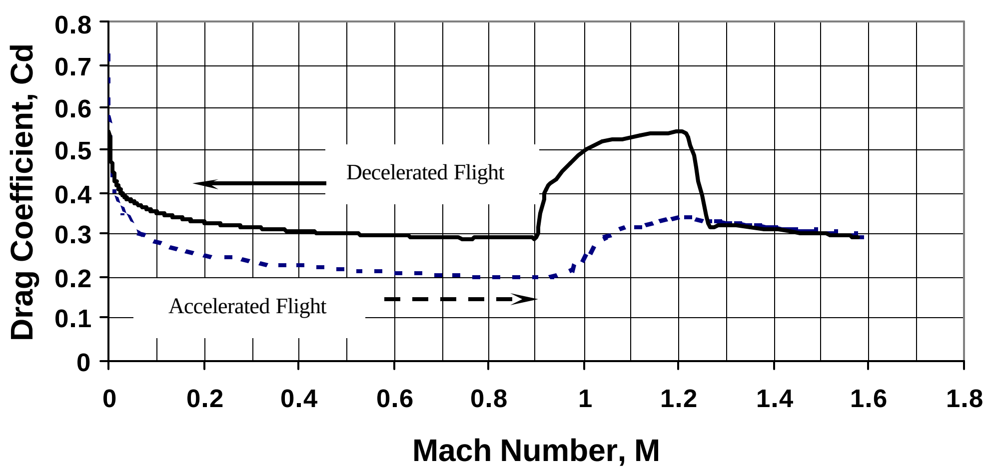
Before we get into the details, it would be wise to discuss some terminology.

Many public or commercially available rocket performance programs require the user to input a value of average drag coefficient 'C_D'. This is often a guess on the part of the user. However, more sophisticated programs attempt to predict the time history of drag, and hence, a more precise time history of the rocket's performance. These programs have their equations embedded in the code, and do not attempt to enlighten the user as to their degree of integrity or sophistication. For those of us who are of a nerdy and curious nature, this just doesn't meet our needs. In fact, some of you may wish to develop your own custom software using the equations provided here.

We will limit our discussion to **Zero Lift Drag** or **Parasite Drag**, which is the total rocket drag independent of lift. This occurs at zero **Angle of Attack** for rockets. **Angle of Attack** refers to the angle of incidence between any lifting component (wing, fin, body, etc.) and its velocity vector. **Induced Drag** is that drag associated with the generation of lift. We will not be dealing with **Induced Drag**. One of the largest contributors to **Zero Lift Drag** over **Subsonic** speeds is **Skin Friction Drag**. **Subsonic** refers to flight speeds well below the speed of sound. **Skin Friction Drag** is the drag resulting from viscous shearing stresses acting over the surface of the rocket. **Form Drag** or **Pressure Drag** is the drag on a body resulting from the summation of the static pressure acting normal to the rocket's surfaces, resolved into the direction opposite of flight. **Base Drag** is a contributor to **Pressure Drag**, and is attributed to the blunt aft end of the rocket. **Base Drag** can be a significant contributor to the rocket's overall drag during power-off flight (after engine burnout). Another contributor to **Pressure Drag** is **Wave Drag**. Wave Drag makes its debut during **Transonic** speeds (about Mach 0.8 to Mach 1.2) and through **Supersonic** speeds (above Mach 1.2). **Wave Drag** is a **Pressure Drag** resulting from static pressure components located to either side of compression or shock waves that do not completely cancel each other. Finally, the last drag contributor we will consider is **Interference Drag**. **Interference Drag** results from two bodies in close proximity, such as fin to body junctures and launch lug to body junctures. Specifically, we will account for the following drag contributors:

- Skin Friction Drag (Viscous Effects)
- Base Drag (Pressure Drag increment due to blunt body during power-off flight)
- Wave Drag (Pressure Drag increment due to compression or shock waves)
- Interference Drag (Drag increment due to bodies in close proximity)

Below is a typical flight history of drag versus Mach number.



In the above graph the rocket accelerates from a standstill on the launch pad to a maximum Mach Number of about 1.6 (1218 mph!). This is depicted by the dashed curve. The rocket then begins to decelerate, still under thrust flight, as depicted by the solid curve. Then at a Mach Number of about 1.25 (951 mph), the engine burn is complete and the rocket continues its deceleration under zero thrust flight. At this point, the solid curve is displaced above the dashed curve, depicting a drag rise due to **Base Drag**. This Base Drag is a result of transitioning from thrust flight to coast or zero thrust flight. The existence of thrust, or lack thereof, <u>can</u> have a significant effect on the rocket's drag coefficient for blunt aft bodies, as in the case above.

Both curves exhibit a jump in drag coefficient over the range of about Mach 0.9 to 1.2. This is the characteristic transonic region where **Wave Drag** makes its debut and may dominate. Again, the shape of the fin and aft body region will have a significant effect on drag rise due to Wave Drag. Wave Drag can be somewhat minimized by tailoring the shape of the fin and aft body geometry using a technique called **Area Ruling**. Although I have used Area Ruling in some of my transonic rocket designs, its overall effect on the rocket's performance is minimal due to the short time spent in transonic flight. For most projects, time spent in efforts such as Area Ruling would yield little return on investment.

There exists a steep increase in drag coefficient as the Mach Number approaches zero. This drag rise is associated with small **Reynolds Numbers.** At very small Reynolds Numbers the momentum of the airflow about the rocket is insufficient to remain attached to its surface and maintain well-defined streamlines, hence the flow separates and becomes turbulent. The result is an increase in **Friction Drag**. The Reynolds Number is the ratio of inertia forces to viscous forces as a vehicle penetrates flow. In mathematical form it is defined by,

$$R_e = \frac{r V_\infty L}{m}$$

Where,

$$\begin{aligned} r &= \textit{Density of fluid} \\ V_\infty &= \textit{Free stream velocity of the fluid about the vehicle} \\ L &= \textit{Characteristic length of vehicle (rocket diameter)} \\ m &= \textit{Absolute coefficient of viscosity} \end{aligned}$$

Reynolds Numbers are commonly used as scaling factors to approximate similar flow conditions in the laboratory (i.e., wind tunnel testing) for situations where it is difficult to recreate actual flow about full scale bodies.

Mach Number is the ratio of the fluid velocity to the speed-of-sound in that fluid. The speed-of-sound in a fluid is the speed at which a pressure disturbance is propagated through the fluid. In air, at Standard Sea Level conditions, the speed of sound is about 762 mph or 1116.4 ft/s. If the fluid velocity about an object is equal to the speed-of-sound of that fluid, then the fluid is said to be travelling at Mach 1.0 relative to that object.

Viscosity is a characteristic of a fluid described by the fluid's ability to resist shear. A fluid having high viscosity will better resist deformation under shear than a fluid having low viscosity. Viscosity of a fluid is often measured by applying a pure torque to the fluid. If one were to integrate the product of shear stresses times the distance to the center of applied torque over the fluid volume, the result would be equal to the applied torque. Shear stress within a fluid is proportional to the gradient of the fluid velocity acting normal to the shear plane. The constant of proportionality is known as the absolute coefficient of viscosity 'm'. For the case of the applied torque, we define the shear stress as:

$$t = m \frac{du}{dr}$$

Where,

$m =$ *Fluid coefficient of viscosity*

$du =$ *Differential of fluid tangential velocity*

$dr =$ *Differential of radial distance from torque center of application*

The **Friction Drag** previously mentioned is directly related to the shear stresses in the fluid. Without viscosity, there would be no shear stres s and likewise no friction drag.

1.0 Friction Drag –

A given rocket's drag will not only be a function of Mach Number, but also altitude. As altitude changes, so does the air viscosity, speed of sound and air density. Viscosity, density and speed of sound will play a role in the equations for drag as well as a strong dependence on Mach Number.

1.1 Body Friction Drag –

The following equations can be solved in sequential order to determine the rocket's body coefficient of drag due to friction.

$a =$ *Speed of sound, fps*

$$= -0.004h + 1116.45, \quad if\ h \le 37000\,ft$$

$$= 968.08, \quad if\ 37000\,ft \le h \le 64000\,ft$$

$$= 0.0007h + 924.99, \quad if\ h \ge 64000\,ft$$

Where,

h = altitude, ft

$n =$ *Kinematic viscosity, ft²/s*

$$= 0.000157e^{ah+b}$$

Where,

a = 0.00002503 and b = 0.0, For h £ 15000 feet

a = 0.00002760 and b = -0.03417, For 15000 £ h £ 30000 feet

a = 0.00004664 and b = -0.6882, For h ³ 30000 feet

$Rn^* =$ *Compressible Reynolds Number*

$$= \frac{aML}{12n}(1 + 0.0283M - 0.043M^2 + 0.2107M^3 - 0.03829M^4 + 0.002709M^5)$$

Where,

M = Mach Number

L = total length of rocket, inches

$Cf^* =$ *Incompressible skin friction coefficient*

$$= 0.037036Rn^{*-0.155079}$$

$Cf =$ *Compressible skin friction coefficient*

$$= Cf*(1+0.00798M-0.1813M^2+0.0632M^3-0.00933M^4+0.000549M^5)$$

$$Cf*(term) = \textit{Incompressible skin friction coefficient with roughness}$$

$$= \frac{1}{\left[1.89+1.62\log_{10}\left(\frac{L}{K}\right)\right]^{2.5}}$$

Where,

K = 0.0, for smooth surface
= 0.00002 to 0.00008, for polished metal or wood
= 0.00016, for natural sheet metal
= 0.00025, for smooth matte paint, carefully applied
= 0.0004 to 0.0012, for standard camouflage paint

$$Cf(term) = \textit{Compressible skin friction coefficient with roughness}$$

$$= \frac{Cf*(term)}{(1+0.2044M^2)}$$

$$Cf(final) = \textit{Final skin friction coefficient}$$

$$= Cf,\ if\ Cf \geq Cf(term)$$

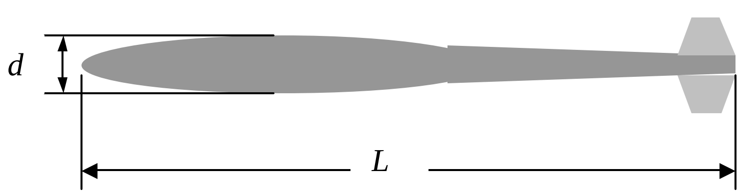
$$= Cf(term),\ if\ Cf \leq Cf(term)$$

$$Cd_f(body) = \textit{Body coefficient of drag due to friction}$$ **Equation 1.1**

$$= Cf(final)\left[1+\frac{60}{(L/d)^3}+0.0025(L/d)\right]\frac{4S_B}{pd^2}$$

Where,

d = maximum body diameter
L = total body length
S_B = total wetted surface area of body



1.2 Fin Friction Drag –

The following equations can be solved in sequential order to determine the rocket's total fin coefficient of drag due to friction.

$$a = \textit{Speed of sound, as defined in Section 1.1- Body Friction Drag}$$

n = *Kinematic viscosity, as defined in Section 1.1 – Body Friction Drag*

Rn^{*} = *Compressible Reynolds Number*

$$= \frac{aMC_r}{12n}(1 + 0.0283M - 0.043M^2 + 0.2107M^3 - 0.03829M^4 + 0.002709M^5)$$

Where,

M = *Mach Number*

C_r = *Root chord of fin, inches*

Cf^{*} = *Incompressible skin friction coefficient*

$$= 0.037036 Rn^{*\,-0.155079}$$

Cf = *Compressible skin friction coefficient*

$$= Cf^{*}(1 + 0.00798M - 0.1813M^2 + 0.0632M^3 - 0.00933M^4 + 0.000549M^5)$$

$Cf^{*}(term)$ = *Incompressible skin friction coefficient with roughness*

$$= \frac{1}{\left[1.89 + 1.62\log_{10}\left(\frac{C_r}{K}\right)\right]^{2.5}}$$

Where,

K = *0.0, for smooth surface*

= *0.00002 to 0.00008, for polished metal or wood*

= *0.00016, for natural sheet metal*

= *0.00025, for smooth matte paint, carefully applied*

= *0.0004 to 0.0012, for standard camouflage paint*

$Cf(term)$ = *Compressible skin friction coefficient with roughness*

$$= \frac{Cf^{*}(term)}{(1 + 0.2044M^2)}$$

$Cf(final)$ = *Final skin friction coefficient*

$$= Cf,\ if\ Cf \geq Cf(term)$$

$$= Cf(term),\ if\ Cf \leq Cf(term)$$

Rn = *Incompressible Reynolds Number*

$$= \frac{aMC_r}{12n}$$

$l = \frac{C_t}{C_r}$ = *Ratio of fin tip chord to root chord*

$$Cf_l = \text{Average flat plate skin friction coefficient for each fin panel}$$

$$= Cf(final)\left[1+\frac{0.5646}{\log_{10}(Rn)}\right],\ if\ \ l = 0.0$$

$$= Cf(final)\frac{[\log_{10}(Rn)]^{2.6}}{(l^2-1)}\left\{\begin{array}{l}\dfrac{l^2}{[\log_{10}(Rn\,l)]^{2.6}}-\dfrac{1}{[\log_{10}(Rn)]^{2.6}}\\ +0.5646\left[\dfrac{l^2}{[\log_{10}(Rn\,l)]^{3.6}}-\dfrac{1}{[\log_{10}(Rn)]^{3.6}}\right]\end{array}\right\}$$

$\longrightarrow$ $Cd_f(fins) = \text{Coefficient of friction drag for all fins}$ **Equation 1.2**

$$= Cf_l\left[1+60\left(\frac{t}{C_r}\right)^4+0.8\left(1+5\overline{X}_{t/c}^{\ 2}\right)\left(\frac{t}{C_r}\right)\right]\frac{4N_f S_f}{pd^2}$$

Where,

t = *Maximum thickness of each fin at root*

C_r =*Fin root chord*

$$\overline{X}_{t/c} = \frac{X_{t/c}}{C_r}$$

$X_{t/c}$ =*Distance from fin leading edge to maximum thickness*

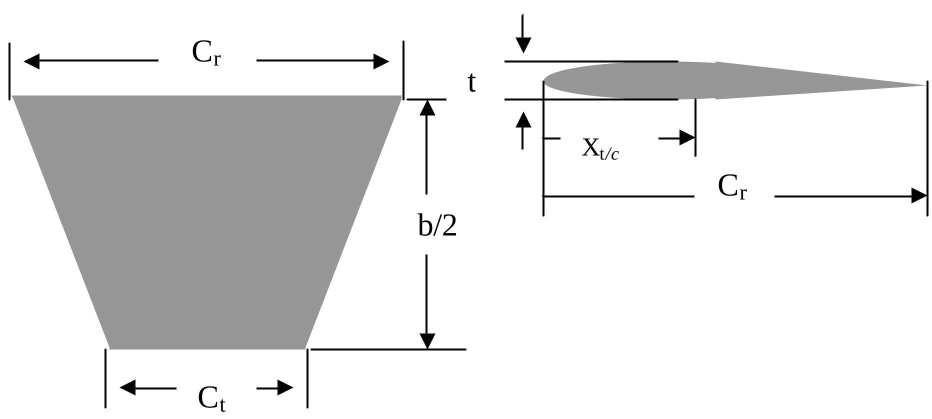
C_t = *Fin tip chord*

N_f = *Number of fins*

S_f = *Total wetted area of each fin*

$$\approx \frac{b}{2}(C_r + C_t)$$

d = *Maximum diameter of rocket body*



1.3 Protuberance Friction Drag –

Protuberances are components that are found on the exterior of a vehicle. An example of a common protuberance for a rocket would be a launch lug. The contribution of a protuberance to drag is often accounted for by its friction drag. Protuberances will also generate an incremental drag rise due to the interaction of their pressure distributions and boundary layers with that of the host body. This incremental drag rise is known as **Interference Drag** and is difficult to predict. A detailed analysis of interference drag is beyond the scope of this text.

The following equations can be solved in sequential order to determine the coefficient of drag due to a protuberance.

$$a = \textit{Speed of sound, as defined in Section 1.1 – Body Friction Drag}$$

$$n = \textit{Kinematic viscosity, as defined in Section 1.1 – Body Friction Drag}$$

$$Rn^* = \textit{Compressible Reynolds Number}$$
$$= \frac{aML_P}{12n}(1 + 0.0283M - 0.043M^2 + 0.2107M^3 - 0.03829M^4 + 0.002709M^5)$$

Where,

M = Mach Number

L_P *= Length of protuberance, inches*

$$Cf^* = \textit{Incompressible skin friction coefficient}$$
$$= 0.037036Rn^{*-0.155079}$$

$$Cf = \textit{Compressible skin friction coefficient}$$
$$= Cf^*(1 + 0.00798M - 0.1813M^2 + 0.0632M^3 - 0.00933M^4 + 0.000549M^5)$$

$$Cf^*(term) = \textit{Incompressible skin friction coefficient with roughness}$$
$$= \frac{1}{\left[1.89 + 1.62\log_{10}\left(\frac{L_P}{K}\right)\right]^{2.5}}$$

Where,

K = 0.0, for smooth surface
= 0.00002 to 0.00008, for polished metal or wood
= 0.00016, for natural sheet metal
= 0.00025, for smooth matte paint, carefully applied
= 0.0004 to 0.0012, for standard camouflage paint

$$Cf(term) = \textit{Compressible skin friction coefficient with roughness}$$
$$= \frac{Cf^*(term)}{(1 + 0.2044M^2)}$$

$$Cf\,(final) = Final\ skin\ friction\ coefficient$$
$$= Cf\,,\ if\ Cf \geq Cf\,(term)$$
$$= Cf\,(term),\ if\ Cf \leq Cf\,(term)$$

$$Cf_{pro} = Friction\ coefficient\ of\ protuberance$$

$$Cf_{pro} = 0.8151 Cf\,(final)\left(\frac{a}{L_p}\right)^{-0.1243}$$

Where,

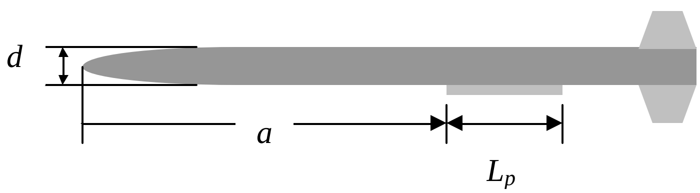
a = Distance from rocket nose to front edge of protuberance
L_p *= Length of protuberance*

$$Cd_{pro} = Drag\ coefficient\ of\ protuberance\ due\ to\ friction$$ **Equation 1.3**

$$= Cf_{pro}\left[1 + 1.798\left(\frac{\sqrt{A}}{L_p}\right)^{3/2}\right]\frac{4S_{pro}}{pd^2}$$

Where,

A = Maximum cross-section area of protuberance
S_{pro} *= Wetted surface area of protuberance*
d = Maximum rocket diameter



1.3 Drag due to Excrescencies –

Excrescencies include features such as scratches, gouges, joints, rivets, cover plates, slots, and holes. These will be accounted for by assuming they are distributed over the wetted surface of the rocket. The coefficient of drag for excrescencies is estimated with the equations below.

$$Cd_e = Change\ is\ drag\ coefficient\ due\ to\ excrescencies$$ **Equation 1.4**

$$Cd_e = K_e \frac{4S_r}{pd^2}$$

Where,

S_r *= Total wetted surface area of rocket*
d = Maximum diameter of rocket body
K_e *= Coefficient for excrescencies drag increment*

$$K_e = 0.00038,\ For\ M < 0.78$$
$$= -0.4501M^4 + 1.5954M^3 - 2.1062M^2 + 1.2288M - 0.26717$$
$$For\ \ 0.78 \leq M \leq 1.04$$
$$= 0.0002M^2 - 0.0012M + 0.0018,\ For\ M > 1.04$$

1.4 Total Friction and Interference Drag Coefficient –

The total friction drag coefficient is assumed proportional to the sum of the drag coefficients for the body, fin, protuberances, and excrescencies. The total skin friction drag coefficient with consideration for interference effects is estimated with the following equation.

$$\longrightarrow Cd_F = \left[Cd_f(body) + K_F Cd_f(fins) + K_F Cd_{pro} + Cd_e\right] \qquad \textbf{Equation 1.5}$$

Where,

$$K_F = Mutual\ interference\ factor\ of\ fins\ and\ launch\ lug\ with\ body$$
$$\approx 1.04$$

2.0 Base Drag Coefficient -

Base drag can be described as a change in mass momentum. Imagine laminar airflow traveling over a smooth gradually contoured body at velocity when suddenly it encounters a blunt aft end where the velocity drops to zero. The mass momentum (mass x velocity) changes abruptly, generating a force that acts opposite to the direction of flight. Fortunately, and particularly in subsonic flow, Mother Nature helps reduce the severity of this change in mass momentum through the generation of a boundary layer. Most likely, the boundary layer is not laminar but turbulent and the momentum thickness is well developed. The change in mass momentum at the blunt end is less severe with the advent of a fully developed boundary layer. The resulting form drag is less severe as well. Unfortunately, nothing is free when it comes to Mother Nature. The boundary layer is developed from the presence of viscosity. Recall that viscosity is the culprit that causes skin friction drag. Generally, as friction drag increases the trend is a reduction in base drag.

Base drag is difficult to predict. An attempt to find an existing method to estimate base drag that correlates well with rocket data was unsuccessful. In lieu of continuing a search, an effort has been made to formulate a method that would estimate base drag with reasonable correlation. The method described below is divided into two regimes, the first for Mach Number less than or equal to 0.6, and the second for Mach Number greater than 0.6.

2.1 Base Drag Coefficient for M < 0.6 –

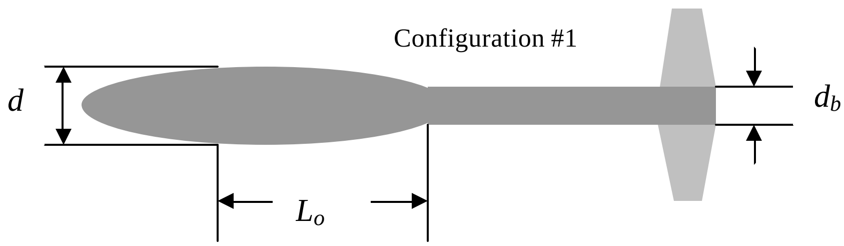
Recall that for coasting flight, as friction drag increases, the tendency is for a reduction in base drag. In fact, base drag is typically described as inversely proportional to the square root of the total skin-friction drag-coefficient. In the formulation given below, it is assumed that the base drag is inversely proportional to the square root of the total skin-friction drag-coefficient, including interference effects. Base drag is also related to the ratio of body base diameter to maximum body diameter. The following is a general form of the equation for base drag.

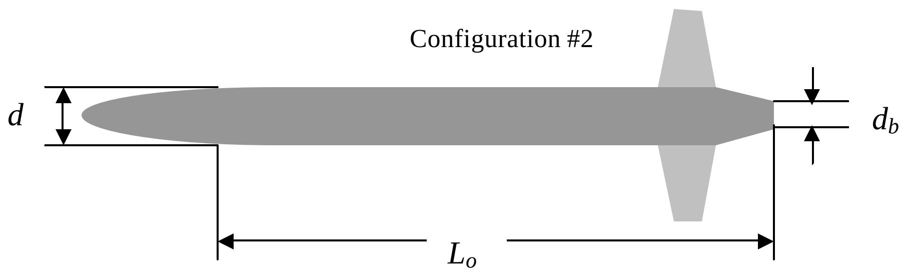
$$\longrightarrow Cd_b(M < 0.6) = K_b \frac{\left(\frac{d_b}{d}\right)^n}{\sqrt{Cd_F}} \qquad \textbf{Equation 1.6}$$

Where,

K_b = *constant of proportionality*
d_b = *base diameter of rocket at aft end*
d = *rocket maximum diameter*
Cd_F = *total skin-friction drag-coefficient, including interference*
n = *exponent*

The values for K_b and n are given as 0.029 and 3.0 in Reference 6. The above equation using these values does a poor job in predicting base drag for the "rocket type" shapes of References 1 and 5. As described in Reference 5, base drag is strongly related to the rocket's length to body ratio, where the length is taken aft of the maximum body-diameter position.



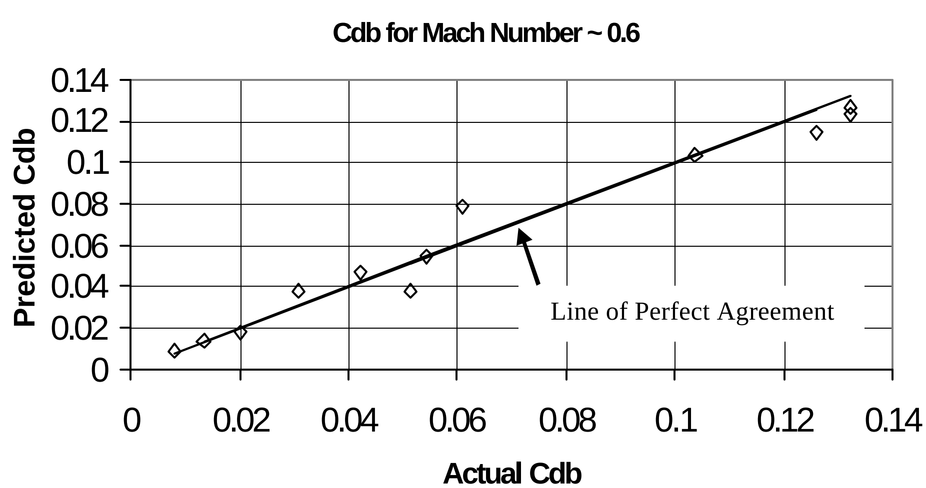


For the above two configurations, reasonable values of K_b and n are:

$$K_b = 0.0274 \tan^{-1}\left[\left(\frac{L_o}{d}\right) + 0.0116\right]$$

$$n = 3.6542\left(\frac{L_o}{d}\right)^{-0.2733}$$

When using the above equations defining K_b and n along with the general equation for $Cd_b(M<0.6)$, the correlation as depicted by the graph below is reasonable.



2.2 Base Drag Coefficient for M > 0.6 –

For Mach Numbers greater than 0.6, the base drag coefficient is calculated relative to the base drag value at Mach = 0.6 (M = 0.6). Using the equation of Section 2.1 to calculate the base drag coefficient at M = 0.6, the base drag coefficient for higher values of Mach Number is determined by multiplying the value at M = 0.6 by the function f_b.

$$\longrightarrow \quad Cd_b(M \geq 0.6) = Cd_b(M = 0.6) f_b \qquad \textbf{Equation 1.7}$$

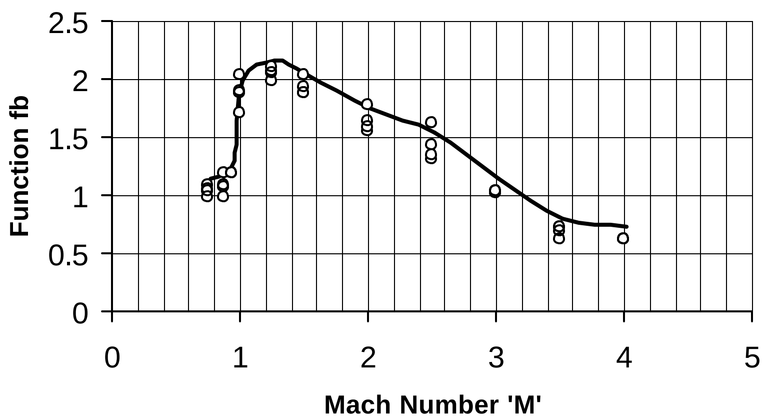
Where,

$$f_b = 1.0 + 215.8(M - 0.6)^{6.0}, \textit{ For } 0.6 < M < 1.0$$

$$f_b = 2.0881(M-1)^3 - 3.7938(M-1)^2 + 1.4618(M-1) + 1.883917, \quad \textit{For } 1.0 < M < 2.0$$

$$f_b = 0.297(M-2)^3 - 0.7937(M-2)^2 - 0.1115(M-2) + 1.64006, \quad \textit{For } M > 2.0$$

The function f_b is based on the sounding rocket data of Reference 1. A plot of f_b against this sounding rocket data is given below.

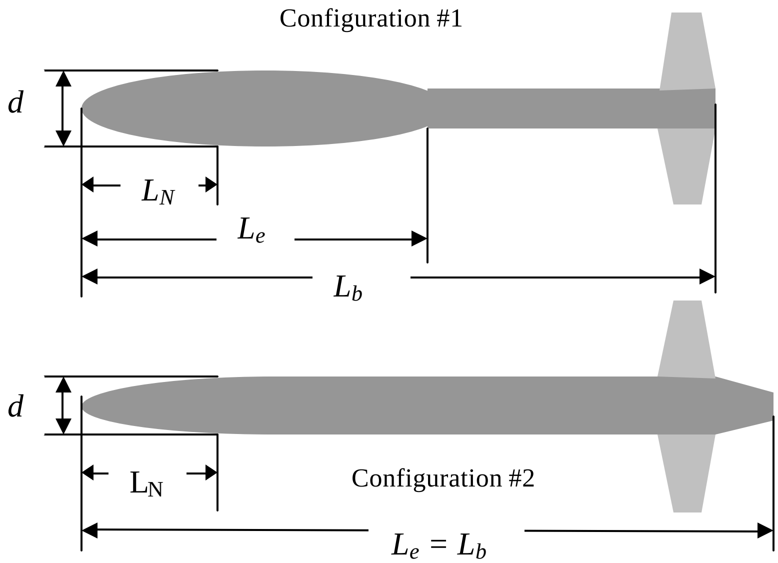


3.0 Transonic Wave Drag Coefficient –

The approach taken was to start with a "clean sheet of paper". The DATCOM methods, as recorded in my hand written notes, proved to predict the transonic drag rise reasonably well for the Hart missile, designation L-65931, of Reference 2. However, the method did not perform as well for a variety of other configurations of References 1 and 5, including the variations of the Hart missile designation L-65930 of References 2 and 3. The depth of the DATCOM methods is beyond the scope of this effort.

The method presented here constitutes a series of equations that characterize the drag rise over the transonic region. These equations are curve fits of actual trend data taken from a variety of rocket configurations, and attempt to predict the drag rise with basic body dimensional data only. Equations based on curve fits of trend data can be dangerous and lead to erroneous results if used outside the range of parameters used in their development. Specifically, the equations presented below should only be used for rockets having a ratio of nose length to effective rocket length less than 0.6. It is guaranteed that the use of these equations for rockets having a ratio greater than 0.6 will result in a truly bad answer. Equations developed this way will tend to lack quantities that relate to the actual physics of the problem, so extrapolation outside the database used in their development is a bad idea. Other methods with more substance, such as DATCOM, may be capable of accounting for the effects of individual component characteristics, such as fin sweep angle.

Drag rise over the transonic region can be predicted for a given Mach Number (M) and basic body dimensions, using the following equations:



$$M_D = \textit{Transonic drag divergence Mach Number}$$
$$= -0.0156\left(\frac{L_N}{d}\right)^2 + 0.136\left(\frac{L_N}{d}\right) + 0.6817$$

Where,

$$L_N = \textit{Length of rocket nose}$$

$$d = \textit{Maximum Body Cross Section Diameter}$$

$$M_F = \textit{Final Mach Number of Transonic Region}$$
$$= a\left(\frac{L_e}{d}\right)^b + 1.0275$$

Where,

$$L_e = \textit{Effective length of rocket}$$

$$a = 2.4, For \left(\frac{L_N}{L_e}\right) < 0.2$$

$$= -321.94\left(\frac{L_N}{L_e}\right)^2 + 264.07\left(\frac{L_N}{L_e}\right) - 36.348, For \left(\frac{L_N}{L_e}\right) \geq 0.2$$

$$b = -1.05, For \left(\frac{L_N}{L_e}\right) < 0.2$$

$$= 19.634\left(\frac{L_N}{L_e}\right)^2 - 18.369\left(\frac{L_N}{L_e}\right) + 1.7434, For \left(\frac{L_N}{L_e}\right) \geq 0.2$$

$$\Delta C_{DMAX} = Maximum\ drag\ rise\ over\ transonic\ region$$

$$= c\left(\frac{L_e}{d}\right)^g, For\ \frac{L_e}{d} \geq 6$$

$$= c(6)^g, For\ \frac{L_e}{d} < 6$$

Where,

$$c = 50.676\left(\frac{L_N}{L_b}\right)^2 - 51.734\left(\frac{L_N}{L_b}\right) + 15.642$$

$$g = -2.2538\left(\frac{L_N}{L_b}\right)^2 + 1.3108\left(\frac{L_N}{L_b}\right) - 1.7344$$

$$\longrightarrow \Delta Cd_T = Transonic\ drag\ rise\ for\ given\ Mach\ Number\ 'M'$$ **Equation 1.8**

$$= \Delta C_{DMAX} F,\ \ If\ M_D \leq M \leq M_F$$

$$= 0., If\ M < M_D\ or\ M > M_F$$

Where,

$$F = -8.3474x^5 + 24.543x^4 - 24.946x^3 + 8.6321x^2 + 1.1195x$$

$$x = \left[\frac{(M - M_D)}{(M_F - M_D)}\right]$$

4.0 Supersonic Wave Drag Coefficient –

For all Mach Numbers greater than M_F, the supersonic drag rise is assumed to equal the transonic drag rise at Mach Number = M_F. This greatly simplifies calculations, and the results compare well with actual test data.

$$\Delta Cd_S = Supersonic\ drag\ rise\ for\ given\ Mach\ Number\ 'M'$$ **Equation 1.9**

$$= \Delta C_{DMAX},\ If\ M \geq M_F$$

$$= 0.,\ If\ M < M_F$$

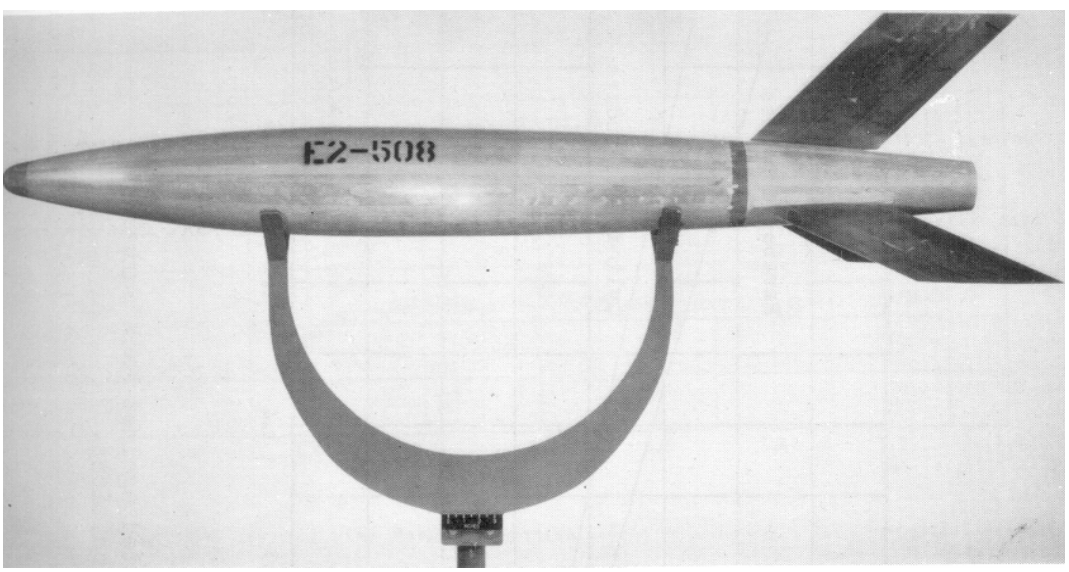
5.0 Total Drag Coefficient –

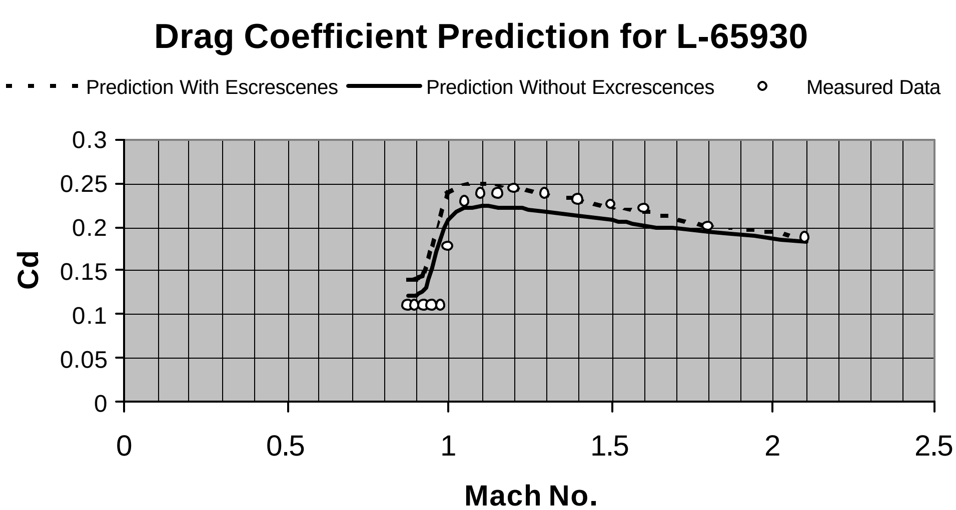
The rocket's total drag coefficient for any given Mach Number is the summation of the individual coefficients given by equations 1.5 to 1.9.

$$C_D = \left[Cd_f(body) + K_F Cd_f(fins) + K_F Cd_{pro} + Cd_e \right] + Cd_b + \Delta Cd_T + \Delta Cd_S$$

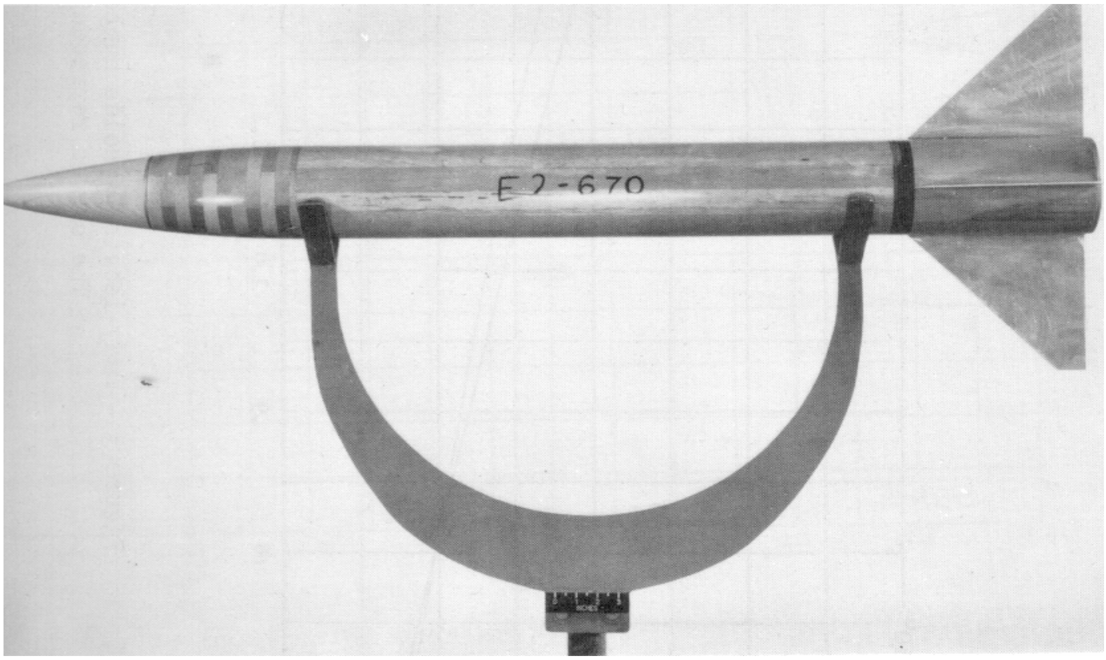
Predictions of drag coefficient versus Mach Number were performed for the two free-flight rocket configurations of Reference 2 and the sounding rocket of Reference 1. In all cases, it was not clear as to the actual surface finish of the rocket. Therefore, the surface finish constant '*K*' was adjusted until the predicted drag coefficients approximated the measured values. Due to the level of sophistication of the equations presented here, excellent agreement between prediction and measured data was not expected. However, it was hoped that prediction of drag magnitude with configuration and variation with Mach Number is reasonable; that is, the trends are correct. For accurate predictions more sophisticated methods such as Finite Difference or Finite Element based Computational Fluid Dynamics should be employed.

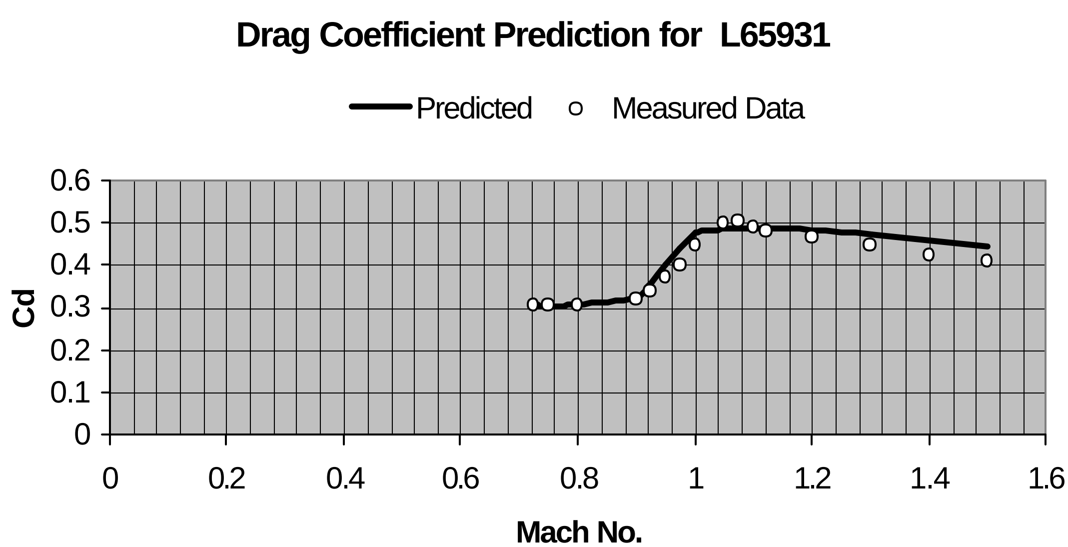
The first example is the rocket configuration L-65930 of NACA TN 3549, Reference 2. The analysis suggested that the rocket surface finish must have been very smooth with very few scratches and/or imperfections.



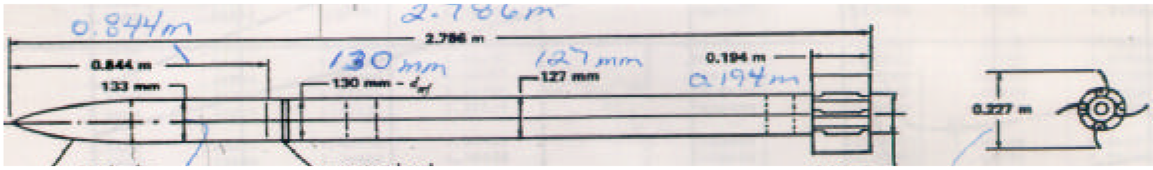


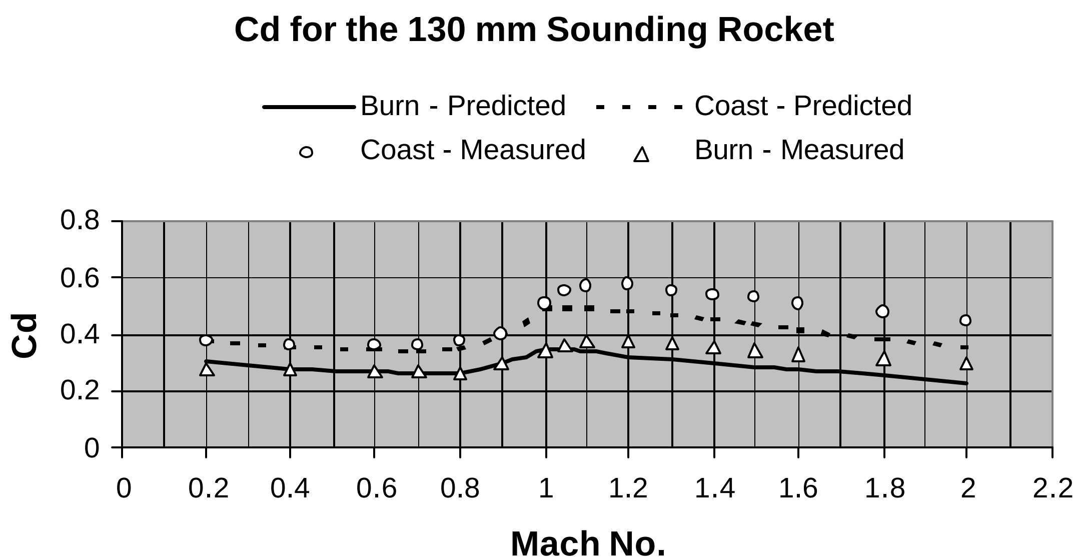
The Second example is the rocket configuration L-65931, also of NACA TN 3549, Reference 2.





The final example is the 130-mm diameter sounding rocket of Reference 1, "How to Make Amateur Rockets". The example compares predicted versus measured drag coefficient for both the thrust (burn) and zero-thrust (coast) phases. Here we can see that the drag rise over the transonic region is under-predicted for both burn and coast flight. The Mach Number at maximum drag rise is under-predicted as well. The geometric configuration of this rocket falls outside the range of data for which the prediction methods were developed. The methods were based on rocket configurations with nose cone length to total rocket length ratios of 0.2 to 0.6. The 130-mm diameter sounding rocket has a ratio of 0.143. The predicted jump from the burn curve to coast curve compares well with that of the measured data over the range of Mach Number.





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